

# Particle Acceleration on the Background of the Kerr-Taub-NUT Spacetime

Changqing Liu<sup>1,2</sup>, Songbai Chen<sup>1</sup>, Chikun Ding<sup>2</sup>, and Jiliang Jing<sup>\*1</sup>

*1) Department of Physics, and Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University,*

*Changsha, Hunan 410081, People's Republic of China and*

*2) Department of Physics and Information Engineering,  
Hunan Institute of Humanities Science and Technology,*

*Loudi, Hunan 417000, P. R. China*

## Abstract

We study the collision of two particles with the different rest masses moving in the equatorial plane of a Kerr-Taub-NUT spacetime and get the center-of-mass (CM) energy for the particles. We find that the CM energy depends not only on the rotation parameter,  $a$ , but also on the NUT charge of the Kerr-Taub-NUT spacetime,  $n$ . Especially, for the extremal Kerr-Taub-NUT spacetime, an unlimited CM energy can be approached if the parameter  $a$  is in the range  $[1, \sqrt{2}]$ , which is different from that of the Kerr and Kerr-Newman black holes.

PACS numbers: 04.70.Dy, 95.30.Sf, 97.60.Lf

Keywords: Particle accelerators, Kerr-Taub-NUT spacetime, center-of-mass energy

---

\* Corresponding author, Email: jljing@hunnu.edu.cn

## I. INTRODUCTION

Recently, Banados, Silk and West (BSW) [1] studied the collision of two particles near a rotating black hole and found that CM energy  $E_{\text{cm}}$  of the two particles moving along the equatorial plane can be arbitrarily high in the limiting case of maximal black hole spin. This means that the extremal rotating black hole could be regarded as a Planck-energy-scale collider, which might bring us some visible signals from ultra high energy collisions, such as, dark matter particles. Thus, the BSW mechanism about the collision of two particles near a rotating black hole has been attracted much attention in the recent years. In Refs. [2, 3] Berti et al pointed out that the arbitrarily high CM energies  $E_{\text{cm}}$  for a Kerr black hole might not be achievable in nature due to the astrophysical limitation, such as the maximal spin [4], gravitational radiation. Subsequently, Lake [5] investigated the CM energy of the collision occurring at the inner horizon of the non-extremal Kerr black hole and found the CM energy is limited [5]. Grib and Pavlov [6–9] argued that the CM energy  $E_{\text{cm}}$  for two particles collision can be unlimited even in the non-maximal rotation if one considers the multiple scattering, and they also evaluated extraction of energy after the collision. The collision in the innermost stable circular orbit was considered in Ref. [10]. The similar BSW mechanism had also been found in other kinds of black holes, e.g. Stringy Black Hole [11], Kerr-Newman black holes [12] and Kaluza-Klein Black Hole [13]. In Refs. [14–16], the author elucidated the universal property of acceleration of particles for rotating black holes and try to give a general explanation of this BSW mechanism for the general rotating black holes. The BSW mechanism stimulated some implications concerning the effects of gravity generated by colliding particles in Ref. [17] and the emergent flux from particle collision near the Kerr black holes [18].

Another interesting stationary axisymmetric object is the Kerr-Taub-NUT (KTN) spacetime [19, 20], which is an important solution of Einstein-Maxwell equations for electro-vacuum spacetime possessing with gravitomagnetic monopole and dipole moments. Besides the mass  $M$  and the rotation parameter  $a$ , the KTN spacetime carries with the NUT charge,  $n$ , which plays the role of a magnetic mass inducing a topology in the Euclidean section. The presence of the NUT charge brings this spacetime some special spacetime structure, such as, Misner singularity. The KNT spacetime serves as an attractive example of spacetimes with asymptotic non-flat structure for exploring various physical phenomena in general relativity

e.g. gravitomagnetism [21]. The KNT solutions representing relativistic thin disks [22] are of great astrophysical importance since they can be used as models for certain galaxies, accretion disks, and the superposition of a black holes and a galaxy or an accretion disk as in the case of quasars. The main purpose of this paper is to study the collision of two particles with the different rest masses in the background of the KTN spacetime and to see what effects of the NUT charge on the CM energy for the particles in the near-horizon collision.

This paper is organized as follows. In Sec. II we derive briefly particle orbits in the KTN spacetime. In Sec. III, we study the collision of two particles with the different rest masses moving in the equatorial plane of the Kerr-Taub-NUT spacetime and discuss the center-of-mass (CM) energy for the particles. Sec.IV is devoted to a brief summary. We use the units  $c = G = 1$  throughout the paper.

## II. PARTICLE ORBITS IN KERR-TAUB-NUT SPACETIME

The metric of the KTN spacetime in the Boyer-Lindquist coordinates can be expressed as [19, 20]

$$ds^2 = -\frac{1}{\Sigma}(\Delta - a^2 \sin^2 \theta)dt^2 + \frac{2}{\Sigma}[\Delta \Xi - a(\Sigma + a\Xi) \sin^2 \theta]dt d\phi \\ + \frac{1}{\Sigma}[(\Sigma + a\Xi)^2 \sin^2 \theta - \Xi^2 \Delta]d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2, \quad (2.1)$$

with

$$\Sigma = r^2 + (n + a \cos \theta)^2, \quad \Delta = r^2 - 2Mr - n^2 + a^2, \quad \Xi = a \sin^2 \theta - 2n \cos \theta, \quad (2.2)$$

where  $M$ ,  $a$  and  $n$  are the mass, the angular parameter and the NUT charge. The radius of the event horizon and the Cauchy horizon of the KTN spacetime are  $r_{H,C} = M \pm \sqrt{M^2 - a^2 + n^2}$ , respectively, which are roots of the equation  $\Delta = 0$ . The existence of the horizons requires  $a^2 \leq M^2 + n^2$ . The angular velocity of the KTN spacetime at the outer horizon is

$$\Omega_H = \frac{a}{2(r_H M + n^2)} = \frac{a}{2(M^2 + n^2 + M\sqrt{M^2 + n^2 - a^2})}. \quad (2.3)$$

In Boyer-Lindquist coordinates system, the timelike and axial Killing vectors are given by  $\xi^a = \left(\frac{\partial}{\partial t}\right)^a$  and  $\psi^a = \left(\frac{\partial}{\partial \phi}\right)^a$ , respectively. With the help of the Killing vectors  $\xi^a$  and  $\psi^a$ ,

we have the following conserved quantities along a geodesic on the equatorial plane

$$E = -g_{ab}\xi^a u^b = -u_t = \frac{r^2 - 2Mr - n^2}{n^2 + r^2} u^t - \frac{a(\Delta - (a^2 + n^2 + r^2))}{r^2 + n^2} u^\phi, \quad (2.4)$$

$$L = g_{ab}\psi^a u^b = u_\phi = \frac{a(\Delta - (a^2 + n^2 + r^2))}{r^2 + n^2} u^t + \frac{(a^2 + n^2 + r^2)^2 - a^2\Delta}{r^2 + n^2} u^\phi, \quad (2.5)$$

where  $u^b$  is the four velocity defined by  $u^b = \frac{dx^b}{d\tau}$ ,  $\tau$  is the proper time for timelike geodesics. Moreover, we can introduce a new conserved parameter  $\kappa$  defined as

$$\kappa = g_{ab}u^a u^b, \quad (2.6)$$

whose values are given by  $\kappa = -1, 0, 1$  corresponding to timelike geodesics, null geodesics, and spacelike geodesics, respectively.

With the help of Eqs. (2.4), (2.5) and (2.6), we can obtain

$$u^t = \frac{1}{(n^2 + r^2)\Delta} \left( E \left\{ (n^2 + r^2)^2 + a^2[3n^2 + r(2M + r)] \right\} - 2aL(n^2 + Mr) \right), \quad (2.7)$$

$$u^\phi = \frac{1}{(n^2 + r^2)\Delta} \left\{ 2aE(n^2 + Mr) + L[-n^2 + r(-2M + r)] \right\}, \quad (2.8)$$

$$u^r = \pm \left[ -\frac{\Delta}{n^2 + r^2} + \frac{1}{(n^2 + r^2)^2} \left\{ E^2[(a^2 + n^2 + r^2)^2 - a^2\Delta] + 2E^2L[-a(a^2 + n^2 + r^2) + a\Delta] - L^2(n^2 - 2Mr + r^2) \right\} \right]^{\frac{1}{2}}, \quad (2.9)$$

where the quantities  $E$  and  $L$  are the specific energy and angular momentum of the particle, respectively. And then the radial equation for the timelike particle moving along geodesics in the equatorial plane is described by

$$\frac{1}{2}u^r u^r + V_{\text{eff}}(r) = 0, \quad (2.10)$$

with the effective potential

$$V_{\text{eff}}(r) = \frac{\Delta}{2(n^2 + r^2)} - \frac{1}{2(n^2 + r^2)^2} \left\{ E^2[(a^2 + n^2 + r^2)^2 - a^2\Delta] + 2E^2L[-a(a^2 + n^2 + r^2) + a\Delta] - L^2(n^2 - 2Mr + r^2) \right\}. \quad (2.11)$$

The circular orbit of the particle is defined by

$$V_{\text{eff}}(r) = 0, \quad \frac{dV_{\text{eff}}(r)}{dr} = 0. \quad (2.12)$$

Since  $u^t > 0$ , the condition

$$E \left\{ (n^2 + r_H^2)^2 + a^2[3n^2 + r(2M + r)] \right\} \geq 2aL(n^2 + Mr), \quad (2.13)$$

must be satisfied. As  $r \rightarrow r_H$  for the timelike particle, this condition reduce to

$$E \geq \frac{aL}{2(n^2 + r_H M)} = \Omega_H L$$

### III. CM ENERGY OF TWO PARTICLES IN THE KERR-TAUB-NUT SPACE-TIME

In this section, we will study the CM energy for the collision of two particles moving in the equatorial plane of the KTN spacetime. Let us now consider two colliding particles with rest masses  $m_1$  and  $m_2$ . We assume that two particles 1 and 2 are at the same spacetime point with the four momenta

$$p_i^a = m_i u_i^a,$$

where  $p_i^a$  and  $u_i^a$  are the four momentum and the four velocity of particle  $i$  ( $i = 1, 2$ ). The sum of the above two momenta is given by

$$p_t^a = p_{(1)}^a + p_{(2)}^a.$$

Then the CM energy  $E_{\text{cm}}$  of the two particles is given by

$$E_{\text{cm}}^2 = -p_t^a p_{ta} = -(m_1 u_{(1)}^a + m_2 u_{(2)}^a)(m_1 u_{(1)a} + m_2 u_{(2)a}). \quad (3.1)$$

Due to  $u^a u_a = -1$ , we obtain

$$\frac{E_{\text{cm}}^2}{2m_1 m_2} = \frac{m_1^2 + m_2^2}{2m_1 m_2} - g_{ab} u_{(1)}^a u_{(2)}^b, \quad (3.2)$$

which can be rewritten as

$$\frac{E_{\text{cm}}}{\sqrt{2m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{2m_1 m_2} + (1 - g_{ab} u_{(1)}^a u_{(2)}^b)}. \quad (3.3)$$

In the case of  $m_1 = m_2$ , the above the equation reduce the result in Refs. [1],[8]. On the background metric (2.1), substitute Eqs. (2.7), (2.8) and (2.9) into Eq. (3.3), the CM energy of two particles in the KTN spacetime is shown by

$$\frac{E_{\text{cm}}}{\sqrt{2m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{2m_1 m_2} + \frac{A(r) - B(r)}{C(r)}}, \quad (3.4)$$

with

$$A(r) = -2a(E_2L_1 + E_1L_2)(n^2 + r) + a^2\left((1 + 3E_1E_2)n^2 + r(r + E_1E_2(2 + r))\right) \quad (3.5)$$

$$- (n^2 + r^2)(n^2 - E_1E_2n^2 + 2r - r^2 - E_1E_2r^2) + L_1L_2(n^2 - (r - 2)r),$$

$$B(r) = \sqrt{b_1(r)b_2(r)}, \quad (3.6)$$

$$b_i(r) = -4aE_iL_i(n^2 + r) + L_i^2(n^2 - (r - 2)r) \quad (3.7)$$

$$+ (n^2 + r^2)\left((1 + E_i^2)n^2 + r(2 + (E_i^2 - 1)r)\right) + a^2\left((3E_i^2 - 1)n^2 + r(E_i^2(2 + r) - r)\right),$$

$$C(r) = (n^2 + r^2)(a^2 - n^2 + (2 - r)r), \quad (3.8)$$

where  $E_i$  and  $L_i$  are the specific energy  $E$  and the angular momentum  $L$  for particle  $i$ . Obviously, the result confirms that the NUT charge  $n$  indeed has influence on the CM energy.

#### A. Near-horizon collision in non-extremal KTN spacetime

We are now in the position to study the properties of the CM energy (3.4) as the radius  $r$  approaches to the horizon  $r_H$  of the non-extremal KTN spacetime. Note that both denominator and the numerator of the fraction  $\frac{A(r)-B(r)}{C(r)}$  on the right-hand side of Eq. (3.4) vanishes at  $r_H$ . Using l'Hospital's rule and taking into account  $r_H^2 - 2r_H - n^2 + a^2 = 0$ , the value of  $E_{\text{cm}}^2$  at  $r_H$  is given by

$$\frac{E_{\text{cm}}^2(r \rightarrow r_H)}{\sqrt{2m_1m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{2m_1m_2} + \frac{A(r)' - B(r)'}{C(r)'}} \bigg|_{r=r_H},$$

with

$$A'(r)|_{r=r_H} = -2a(E_2L_1 + E_1L_2) + 2(E_1E_2 - 1)n^2 - 2L_1L_2(r_H - 1)$$

$$+ 4E_1E_2r_H + 2(1 + 3E_1E_2)n^2r_H + 2(E_1E_2 - 1)r_H^2 + 2(1 + E_1E_2)r_H^3,$$

$$b'_i(r)|_{r=r_H} = -4aE_iL_i + 2(1 + E_i^2)n^2 - 2L_i^2(r_H - 1)$$

$$+ 4E_ir_H + 2(3E_i^2 - 1)n^2r_H + 2(1 + E_i^2)r_H^2 + 2(E_i^2 - 1)r_H^3,$$

$$C'(r)|_{r=r_H} = 2(r_H - 1)(n^2 + r_H^2),$$

$$B'(r)|_{r=r_H} = B \frac{1}{2} \left( \frac{b'_1}{b_1} + \frac{b'_2}{b_2} \right).$$

By implementing the calculation and taking the limit, we reach

$$\frac{E_{\text{cm}}(r \rightarrow r_H)}{2\sqrt{m_1 m_2}} = \left\{ \frac{(m_1 - m_2)^2}{4m_1 m_2} + 1 + \frac{[(L_{H1} - L_1) - (L_{H2} - L_2)]^2 + (L_{H1} L_2 - L_{H2} L_1)^2 \frac{2(n^2+1)-a^2-2\sqrt{1+n^2-a^2}}{4(n^2+1+\sqrt{1+n^2-a^2})^2}}{4(L_{H1} - l_1)(L_{H2} - L_2)} \right\}^{\frac{1}{2}}. \quad (3.9)$$

This is the formula for the CM energy of two particles along the general geodesic orbits on the outer horizon. The critical angular momenta  $L_{Hi}$  ( $i = 1, 2$ ) can be written as  $L_{Hi} = \frac{E_i}{\Omega_H} = \frac{2E_i(n^2+r_H)}{a}$ . In fact, as we will prove in section III B, Eq. (3.9) is valid even in the extremal KTN spacetime simply by taking the near-extremal limit  $a \rightarrow \sqrt{1+n^2}$ . The necessary condition for obtaining an arbitrarily high  $E_{\text{cm}}$  is therefore  $L \approx L_H$  or  $\Omega_H L \approx E$  for either of the two particles. For  $E_1 = E_2 = E$ , we denote  $L_{H1} = L_{H2} = L_H = \frac{2(n^2+r_H)}{a}$  and Eq. (3.9) reduces to

$$\frac{E_{\text{cm}}(r \rightarrow r_H)}{2\sqrt{m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{4m_1 m_2} + 1 + \frac{(L_1 - L_2)^2 L_H}{4(L_H - L_1)(L_H - L_2)a}}. \quad (3.10)$$

When NUT charge vanishes ( $n = 0$ ), the above equation reduces to the result in Ref. [9] for the Kerr black hole. As mentioned in Ref. [12], to obtain an arbitrarily high CM energy  $E_{\text{cm}}$ , one of the colliding particles should have critical angular momentum  $L_H$ . We assume that  $L_1 \rightarrow L_H$  and obtain

$$\frac{E_{\text{cm}}}{2\sqrt{m_1 m_2}} \simeq \sqrt{\frac{(L_2 - L_H)L_H}{4(L_1 - L_H)a}}. \quad (3.11)$$

Now we will discuss how the rotating parameter  $a$ , and the NUT charge  $n$  affect the CM energy  $E_{\text{cm}}$ . We denote the small parameter  $\xi = a_{\text{max}} - a$  with  $a_{\text{max}} = \sqrt{1+n^2}$ . For fixed NUT charge  $n$  and  $\xi$ , the range  $[L_{\text{min}}, L_{\text{max}}]$  of angular momentum for the particles to reach the horizon can be determined numerically with the effective potential  $V_{\text{eff}}$  for the near-extremal KTN spacetime. For an angular momentum  $L \in [L_{\text{min}}, L_{\text{max}}]$ , we can get a negative  $V_{\text{eff}}(l)$  for  $r > r_H$ . However, for arbitrary charge  $n$  and  $\xi$ , we find that, within a small range near the horizon  $r_H$ , the effective potential  $V_{\text{eff}}(L_H)$  is always positive, which means the angular momentum  $L_H$  does not lie in the range  $[L_{\text{min}}, L_{\text{max}}]$ . So the CM energy  $E_{\text{cm}}$  in (3.10) is not divergent. Thus, the CM energy is finite for arbitrary charge  $n$  and rotating parameter  $a$ . Considering that one of the colliding particles has the maximum angular momentum  $L_{\text{max}}$  and another one has the minimum angular momentum  $L_{\text{min}}$ , we obtain the CM energy per unit rest mass for different  $n$  and  $\xi$ . The result is shown in Table

TABLE I: The CM energy per unit rest mass  $\frac{E_{\text{cm}}}{m_1}$  for the KTN spacetime with rotating parameter  $a = a_{\text{max}} - \xi$ ,  $m_1 = m_2 = 1$  and  $L_1 = L_{\text{max}}$ ,  $L_2 = L_{\text{min}}$ .

|         | $\xi=0.1$ | $\xi=0.01$ | $\xi=0.001$ | $\xi=0.0001$ |
|---------|-----------|------------|-------------|--------------|
| $n=0$   | 6.901     | 12.5354    | 22.6352     | 40.4856      |
| $n=0.2$ | 6.8814    | 12.4733    | 22.2955     | 39.6429      |
| $n=0.4$ | 6.8248    | 12.1373    | 21.4312     | 38.2908      |
| $n=0.6$ | 6.7369    | 11.7017    | 19.8102     | 36.3196      |
| $n=0.8$ | 6.6546    | 11.3053    | 19.1093     | 33.1950      |

I. From the table, we can see that, for the KTN spacetime with rotating parameter  $a$  less than  $a_{\text{max}}$  there will be an upper bound for the CM energy. It is also suggested that the CM energy grows very slowly as the maximally spinning case ( $\xi \rightarrow 0$ ) is approached. For fixed parameter  $\xi$ , the value of CM energy decreases with the increase of the charge  $n$ . For the case  $n = 0$ , it reduces to result for the Kerr black hole and we recover the numerical result in Ref. [3].

### B. Near-horizon collision in the extremal KTN spacetime

For the extremal KTN spacetime, the rotating parameter  $a$  and NUT charge  $n$  satisfy the relation  $n^2 = a^2 - 1$ , the numerator  $A(r) - B(r)$  and the denominator  $C(r)$  both must have a second-order zero at  $r = r_H$ . Using l'Hospital's rule twice, we obtain

$$\frac{E_{\text{cm}}(r \rightarrow r_H)}{2\sqrt{m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{4m_1 m_2} + \frac{A(r)'' - B(r)''}{C(r)''}} \Big|_{r=r_H}, \quad (3.12)$$

where the second-order derivatives are given by

$$\begin{aligned} A''(r)|_{r=r_H} &= -2L_1 L_2 + 2(1 + 3E_1 E_2)n^2 + 2r_H(-4 + 5r_H + E_1 E_2(2 + 5r_H)) \\ b_i''(r)|_{r=r_H} &= -2L_i^2 + 2(3E_i^2 - 1)n^2 + 2r_H(4 - 5r_H + E_i^2(2 + 5r_H)) \\ C''(r)|_{r=r_H} &= 2(n^2 + r_H(5r_H + 4)) \\ B''(r)|_{r=r_H} &= B \left[ \frac{1}{2} \left( \frac{b_1''}{b_1} + \frac{b_2''}{b_2} \right) - \frac{1}{4} \left( \frac{b_1'}{b_1} - \frac{b_2'}{b_2} \right)^2 \right]. \end{aligned}$$



After tedious calculation, we obtain the CM energy at the outer horizon for the extremal KTN spacetime

$$\frac{E_{\text{cm}}(r \rightarrow r_H)}{2\sqrt{m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{4m_1 m_2} + 1 + \frac{[(L_{H1} - L_1) - (L_{H2} - L_2)]^2 + \frac{(L_{H1}L_2 - L_{H2}L_1)^2}{4a^2}}{4(L_{H1} - l_1)(L_{H2} - L_2)}}. \quad (3.13)$$

For the case  $E_1 = E_2 = 1$ , Eq. (3.13) reduces to

$$\frac{E_{\text{cm}}(r \rightarrow r_H)}{2\sqrt{m_1 m_2}} = \sqrt{\frac{(m_1 - m_2)^2}{4m_1 m_2} + \frac{1}{2} \left( \frac{L_1 - \check{L}_H}{L_2 - \check{L}_H} + \frac{L_2 - \check{L}_H}{L_1 - \check{L}_H} \right)}. \quad (3.14)$$

where the critical angular momentum  $\check{L}_H = 2a$ . For the special case  $m_1 = m_2$  and  $n = 0$ , the above equation recovers the result obtained by BSW [1]. The expression (3.14) shows that the unlimited CM energy can be approached if one of the colliding particles has critical angular momentum  $\check{L}_H$  which ensures the particle can reach the outer horizon. Since the particles move along the stable circular orbits in the equatorial plane, there must exist a restriction for the angular momentum, which is also shown in Table II.

TABLE II: The ranged angular momentum  $L$  for the extremal KTN spacetime with different rotating parameter  $a$  and NUT charge  $n$ .

|                  | $a=1$  | $a=1.1$ | $a=1.2$ | $a=\sqrt{2}$ | $a=1.8$ |
|------------------|--------|---------|---------|--------------|---------|
| $L_{\text{max}}$ | 2      | 2.048   | 2.350   | 2.828        | 3.536   |
| $L_{\text{min}}$ | -4.828 | -5.027  | -5.222  | -5.633       | -6.353  |

With the help the effective potential (2.11), we can determine the range of the rotating parameter  $a$  for the extremal KTN spacetime. The effective potential for a particle with critical angular momentum  $\check{L}_H$  is

$$V_{\text{eff}(L \rightarrow \check{L}_H)} = -\frac{(r-1)^2(r+1-a^2)}{(r^2+a^2-1)^2}. \quad (3.15)$$

As expected, the effective potential  $V_{\text{eff}(L \rightarrow \check{L}_H)}$  approaches 0 at spatial infinity. Obviously, the condition for the particle falling freely from rest at infinity to the horizon can be expressed as

$$V_{\text{eff}(L \rightarrow \check{L}_H)} \leq 0 \quad \text{for any } r \geq 1. \quad (3.16)$$

Combing with the condition  $1 + n^2 \geq a^2$ , we can solve Eq. (3.15) and obtain the range for the parameters  $a$  and  $n$

$$1 \leq a \leq \sqrt{2}, \quad |n| \leq 1, \quad (3.17)$$

which means that for the KTN extremal spacetime with  $a \in [1, \sqrt{2}]$  and the NUT charge  $|n| \leq 1$ , the particle with critical angular momentum  $\check{L}_H$  can reach the horizon. Thus, for the fixed  $a \in [1, \sqrt{2}]$ , one can find that the CM energy will be unlimited if  $L_1 = \check{L}_H$  and  $L_2$  is in a proper range. In Fig.1, we plot the effective potential  $V_{\text{eff}(L \rightarrow \check{L}_H)}$  and CM energy  $E_{\text{cm}}$  of collision for the different values of  $a$  and  $n$ . From Fig. 1 (a), one can find that for  $1 \leq a \leq \sqrt{2}$  the effective potential  $V_{\text{eff}(L \rightarrow \check{L}_H)}$  is negative when  $r \geq r_H = 1$  so that the particle can reach the horizon. However, the effective potential  $V_{\text{eff}(L \rightarrow \check{L}_H)}$  for  $a = 1.8$  is positive near the horizon, which implies that the particle can not reach the horizon in this case. From Fig.1 (b), we also find that for the case  $1 \leq a \leq \sqrt{2}$  the CM energy at the horizon is be unlimited, which can be explained by a fact that the effective potential  $V_{\text{eff}(L \rightarrow \check{L}_H)}$  is negative for the two colliding particles with angular momenta  $L_1 = \check{L}_H$  and  $L_2 = -2$ . For the case  $a = 1.8$ , it is obvious that the CM energy is limited and the particle cannot reach the horizon. Moreover, we also find that with the increase of the NUT charge  $n$  the CM energy increases, but the rate of increase of the CM energy decreases in the KTN extremal spacetime.

Now we would like to estimate the maximal value of the CM energy of the collision particles in the background of the extremal KTN spacetime. Here, we consider the case the angular momentum of one of the particles deviates little from the critical angular momentum  $\check{L}_H$ . For simplicity, we choose  $L_1 = \check{L}_H - \delta L$  and  $L_2 = 0$ . From Eq. (3.14), we obtain that the maximal CM energy for the collision particles can be approximated as

$$\frac{E_{\text{cm}}^{\text{max}}}{\sqrt{m_1 m_2}} \approx \sqrt{2a} \delta L^{-1/2} + O(\delta L^{1/2}) = \sqrt{2\sqrt{1+n^2}} \delta L^{-1/2} + O(\delta L^{1/2}), \quad (3.18)$$

in the extremal KTN spacetime. Clearly, the maximal CM energy increases with the increase of NUT charge. Now, we also estimate the maximal value of the CM energy of particles in the case of the near-extremal case. Here we denote a small deviation  $\xi = a_{\text{max}} - a \ll 1$ , and suppose that  $L_1 = \check{L}_H$  and  $L_2 = 0$ . Then, it is easy to obtain from Eq. (3.10) that the maximal CM energy of the collision particles can be estimated by

$$\frac{E_{\text{cm}}^{\text{max}}}{\sqrt{m_1 m_2}} \approx 2\sqrt{\frac{a^3}{8}} \delta \xi^{-1/4} + O(\delta \xi^{1/4}), \quad (3.19)$$

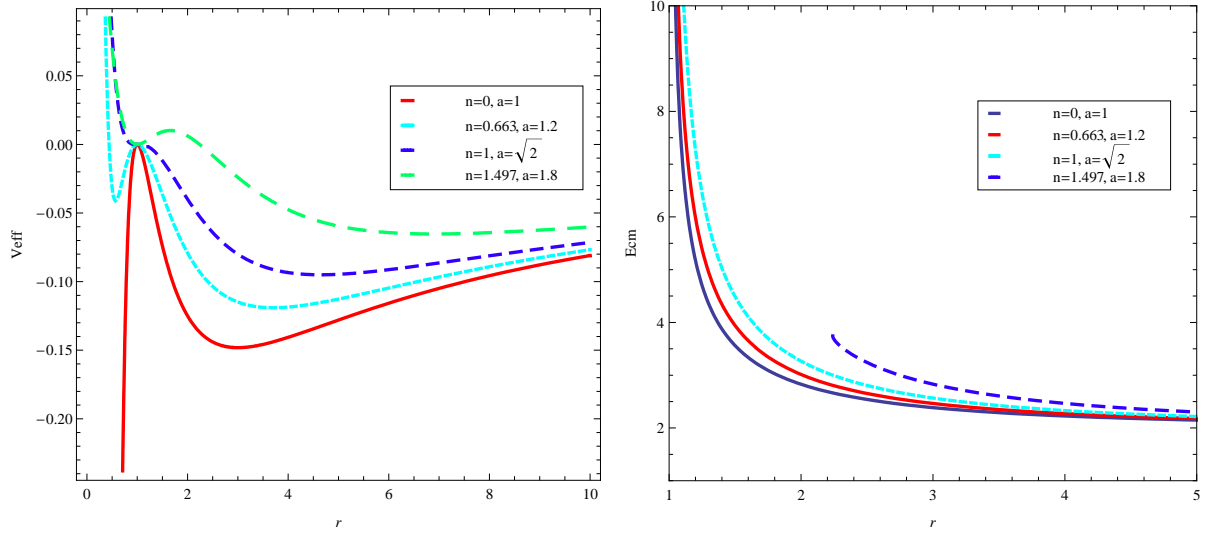


FIG. 1: (Color online) For an extremal KTN spacetime (a) the variation of the effective potential  $V_{\text{eff}(L \rightarrow \check{L}_H)}$  with radius vs  $a$  with angular momentum  $L = \check{L}_H = 2a$ . (b) the variation of the CM energy  $E_{\text{cm}}$  with radius vs  $a$  with  $m_1 = m_2 = 1$  and  $L_1 = \check{L}_H, l_2 = -2$ .

in the near-extremal KTN spacetime. This means that the maximal CM energy of the collision particles increases proportionally to  $a^{3/4}$ . We assume that the rest masses of the colliding particles  $m_1, m_2$  are of about 1 GeV, just like the mass of a neutron. In order to obtain the Planck-scale energy  $E_{\text{cm}} \sim 10^{19}$  GeV, we need  $\delta\xi \sim 10^{-76}$ , which is similar to that in the Kerr-Newman black hole [12]. This implies that it is very hard for a near-extremal case to be a particle accelerator of Planck-scale energy.

#### IV. SUMMARY

In this paper, we studied the collision of two particles with the different rest masses moving in the equatorial plane of the KTN spacetime and get the center-of-mass (CM) energy for the particles. Our result shows that the CM energy depends not only on the rotation parameter of the KTN spacetime,  $a$ , but also on the NUT charge of the KTN spacetime,  $n$ . For the extremal KTN spacetime, the presence of the NUT charge modified the restrict conditions for the spin  $a$  when arbitrarily high CM energy appears, i.e.,  $1 \leq a \leq \sqrt{2}$ , which is a significant difference from the Kerr [1] and Kerr-Newman [12] black holes. For the near-extremal case, we also found that the CM energy  $E_{\text{cm}}$  decreases with the increase of the

NUT charge  $n$  when one particle has the maximum angular momentum  $L_{\max}$  and the other has the minimum angular momentum  $L_{\min}$ . We also estimated the maximal value of the CM energy of particles for both the extremal and the near-extremal KTN spacetime when one particle has the critical angular momentum and the other has zero angular momentum and discussed the change of the maximal CM energy with the parameters  $a$  and  $n$ .

### Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No 10875040, 10875041; the key project of the National Natural Science Foundation of China under Grant No 10935013; the National Basic Research of China under Grant No. 2010CB833004, PCSIRT under Grant No. IRT0964, and the Construct Program of the National Key Discipline.

- 
- [1] M. Ban ados, J. Silk and S. M. West, Phys. Rev. Lett. **103**, 111102 (2009) [arXiv:0909.0169 [hep-ph]].
  - [2] E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius and U. Sperhake, Phys. Rev. Lett. **103**, 239001 (2009) [arXiv:0911.2243 [gr-qc]].
  - [3] T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. **104**, 021101 (2010) [arXiv:0911.3363 [gr-qc]].
  - [4] K. S. Thorne, Astrophys. J. **191**, 507 (1974).
  - [5] K. Lake, Phys. Rev. Lett. **104**, 211102 (2010) [Erratum-ibid. **104**, 259903 (2010)] [arXiv:1001.5463 [gr-qc]].
  - [6] A. A. Grib and Yu. V. Pavlov, Astropart. Phys. **34**, 581 (2011) [arXiv:1001.0756 [gr-qc]].
  - [7] A. A. Grib and Yu. V. Pavlov, [arXiv:1004.0913 [gr-qc]].
  - [8] A. A. Grib and Yu. V. Pavlov, [arXiv:1007.3222 [gr-qc]].
  - [9] A. A. Grib and Yu. V. Pavlov, Grav. Cosmol. **17**, 42 (2011) [arXiv:1010.2052[gr-qc]].
  - [10] T. Harada and M. Kimura, Phys. Rev. D **83**, 024002 (2011). [arXiv:1010.0962 [gr-qc]].
  - [11] S. W. Wei, Y. X. Liu, H. T. Li and F. W. Chen, JHEP **1012**, 066 (2010) [arXiv:1007.4333 [hep-th]].
  - [12] S. W. Wei, Y. X. Liu, H. Guo and C. E. Fu, Phys. Rev. D **82**, 103005 (2010) [arXiv:1006.1056 [hep-th]].
  - [13] P. J. Mao, L. Y. Jia, J. R. Ren and R. Li, [arXiv:1008.2660 [hep-th]].
  - [14] O. B. Zaslavskii, Phys. Rev. D **82**, 083004 (2010) [arXiv:1007.3678 [gr-qc]].
  - [15] O. B. Zaslavskii, JETP Letters. **92**, 570 (2010) [arXiv:1007.4598v1 [gr-qc]].

- [16] O. B. Zaslavskii, *Class.Quant.Grav.***28**, 105010 (2011) [arXiv:1011.0167 [gr-qc]].
- [17] M. Kimura, K. Nakao, and H. Tagoshi, *Phys.Rev.D* **83**, 044013 (2011) [arXiv:1010.5438[astro-ph.CO]].
- [18] M. Banados, B. Hassanain, J. Silk and S. M. West, *Phys. Rev. D* **83**, 023004 (2011) [arXiv:1010.2724 [astro-ph.CO]].
- [19] M. Demiański and E. T. Newman, *Bulletin de l'Académie Polonaise des Sciences* **XIV**, 653 (1966).
- [20] J. G. Miller, *J. Math. Phys.* **14**, 486 (1973).
- [21] D. Bini, C. Cherubini, R. T. Jantzen, and B. Mashhoon, *Phys. Rev. D* **67**, 084013 (2003).
- [22] G. Garca-Reyes and G. A. Gonzlez, *Phys. Rev. D* **70**, 104005 (2004).